Roper Properties on the Lattice: An Update

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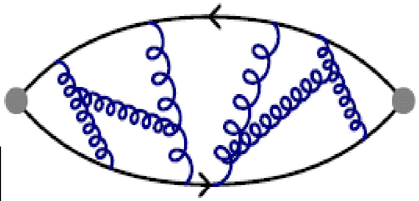
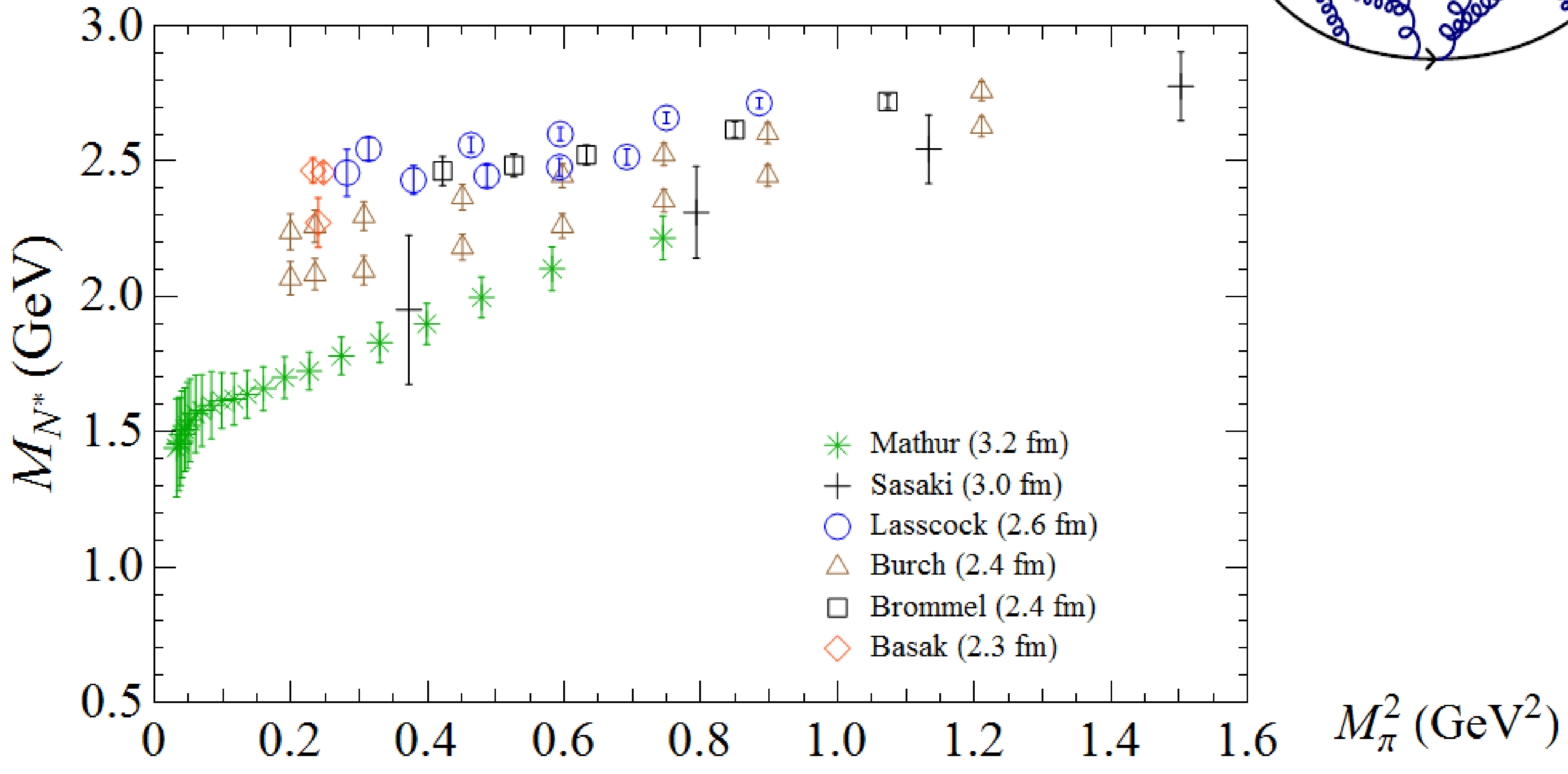
Resonances

Last Nstar

§ Quenched calculation (statistical errorbars)



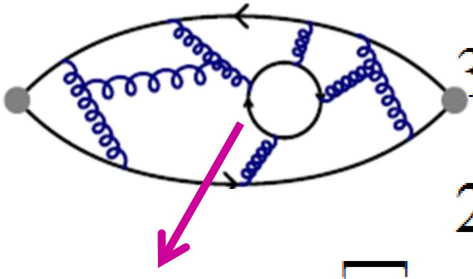


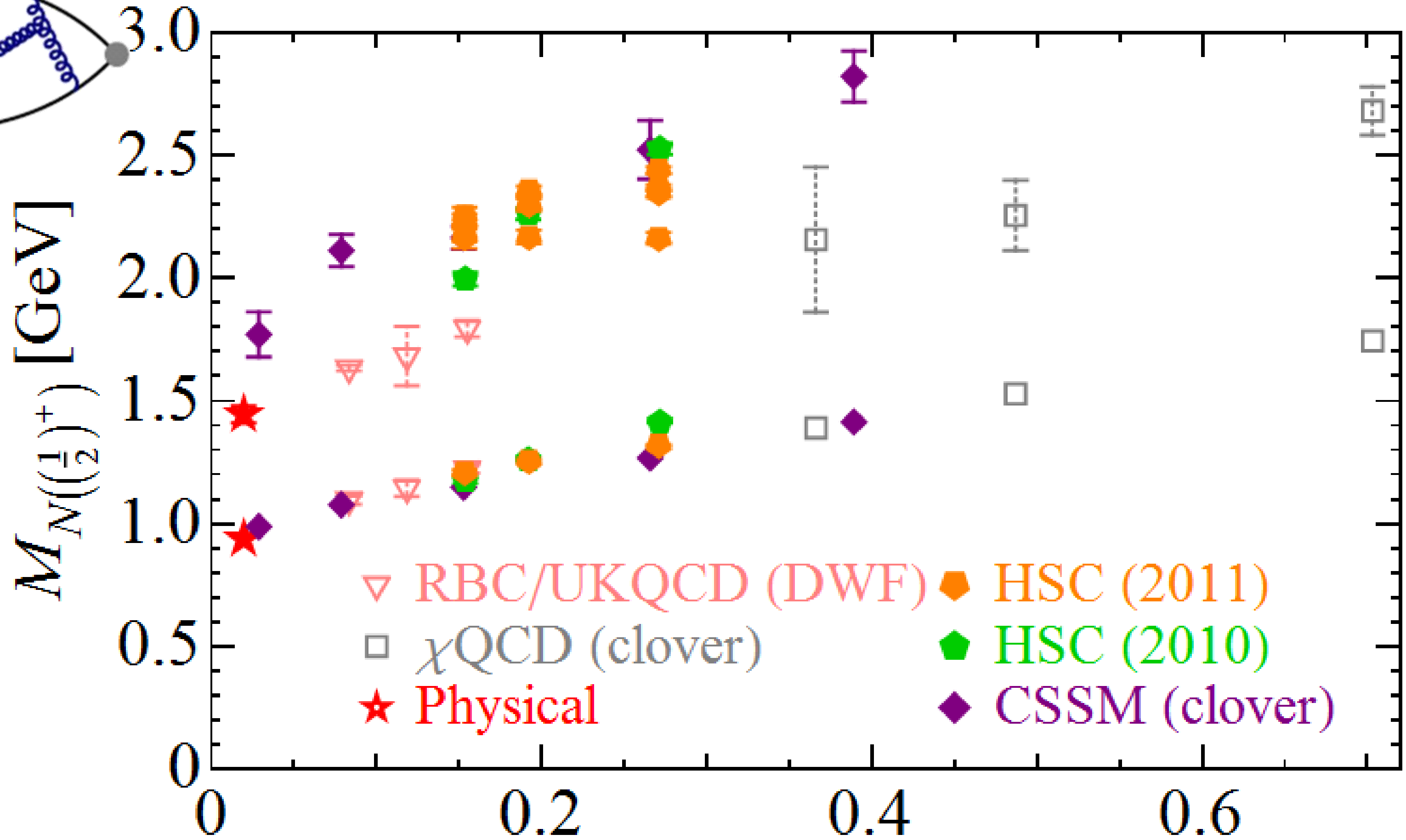
§ Systematics: Finite-volume effect, sea N_f, lattice spacing, etc.

N_f=2+1 Roper

§ More dynamical results (statistical errorbars)

S. Wallace, R. Edwards, D. Leinweber, this workshop

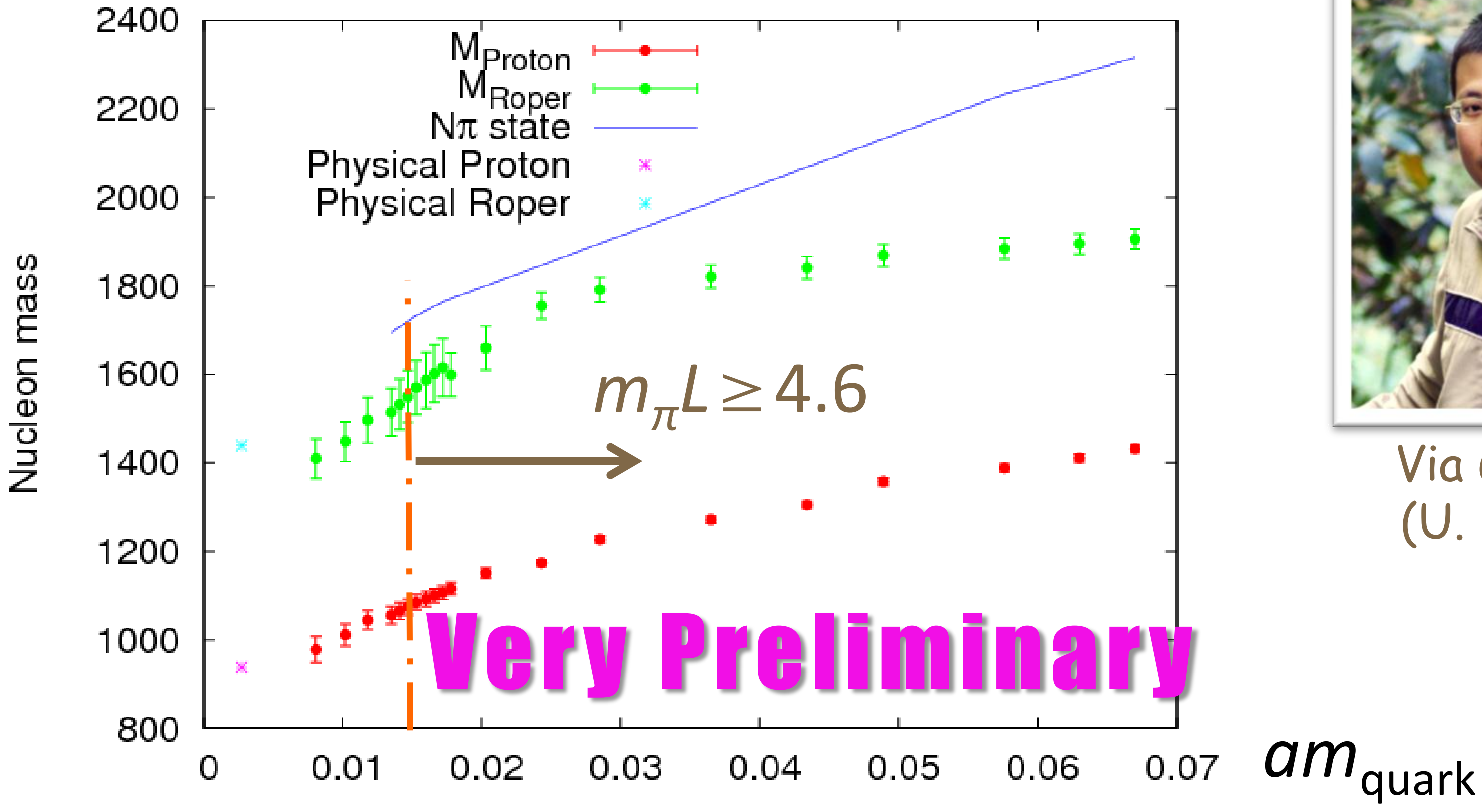




$\mathcal{N}_f$=2+1 Roper

§ χQCD Collaboration (partially quenched)

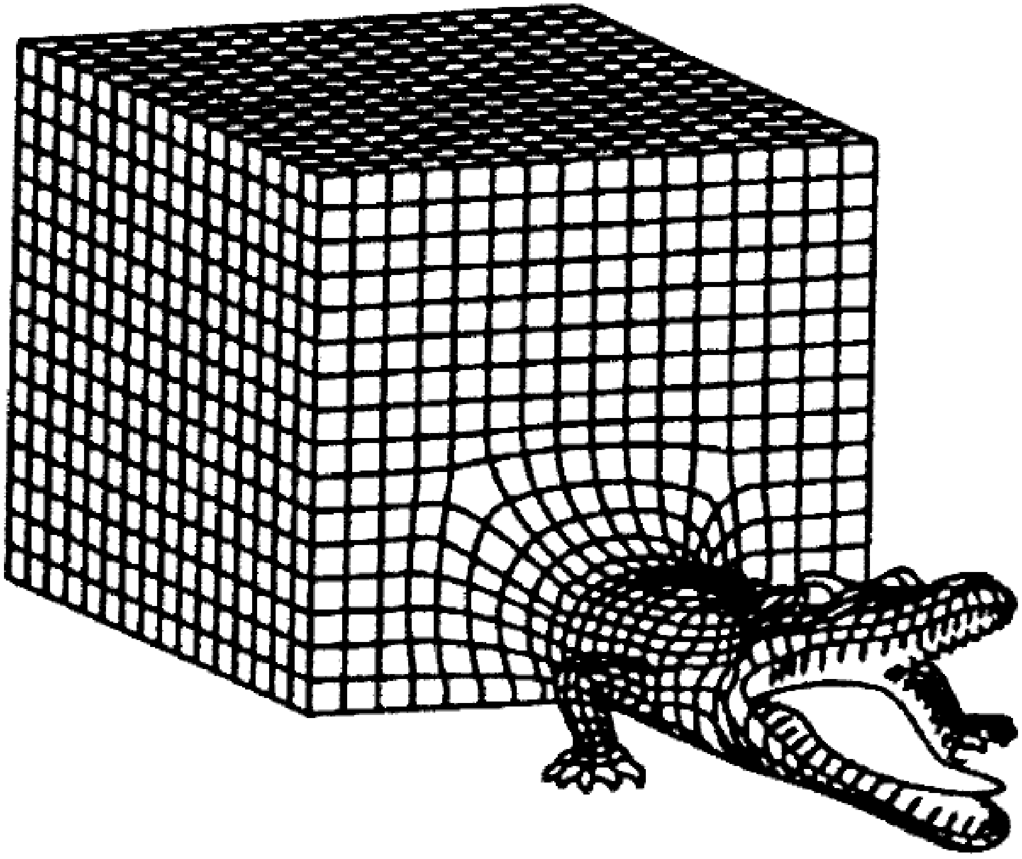
- Used fermions that preserve chiral symmetry at finite *a*
- Overlap on DWF (with PQ m_π as light as 100 MeV)



Via Gong Ming
(U. Kentucky)

"Welcome to the lattice and its dangerous animals."

Karl Jansen



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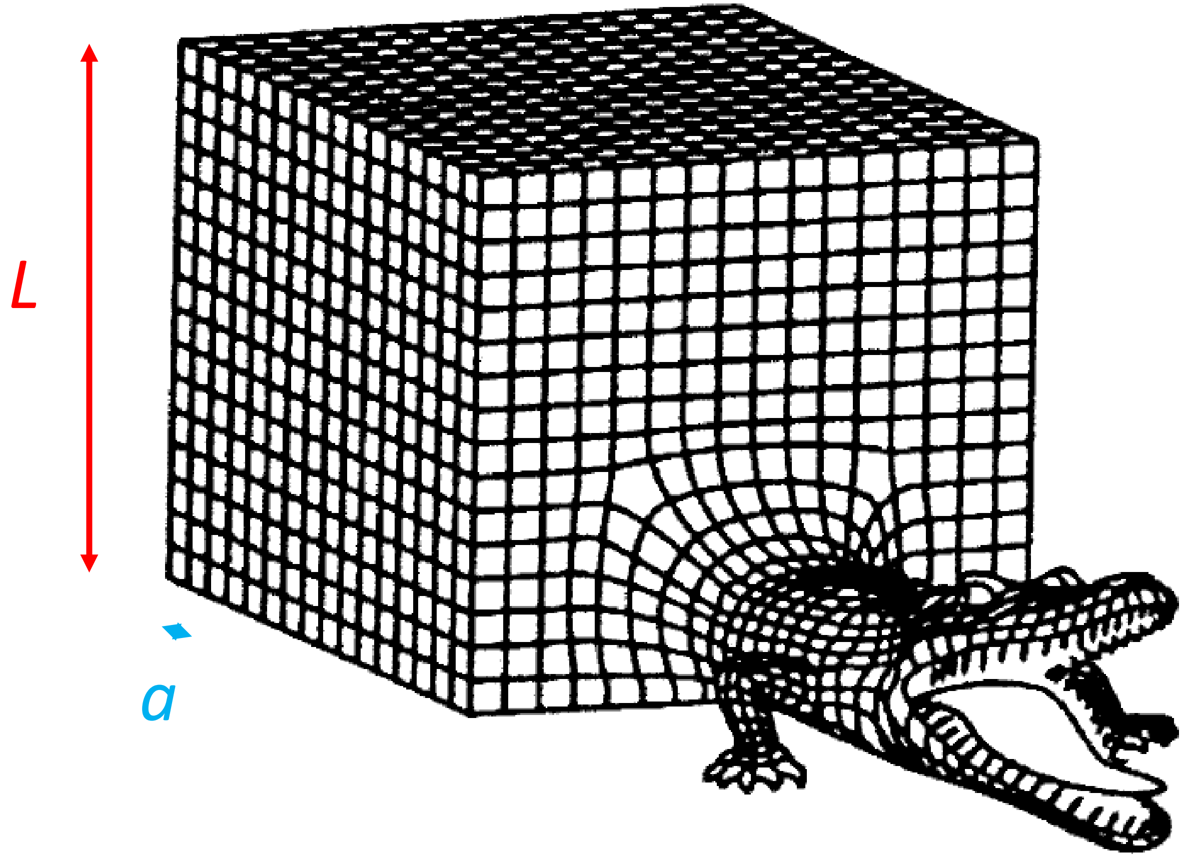
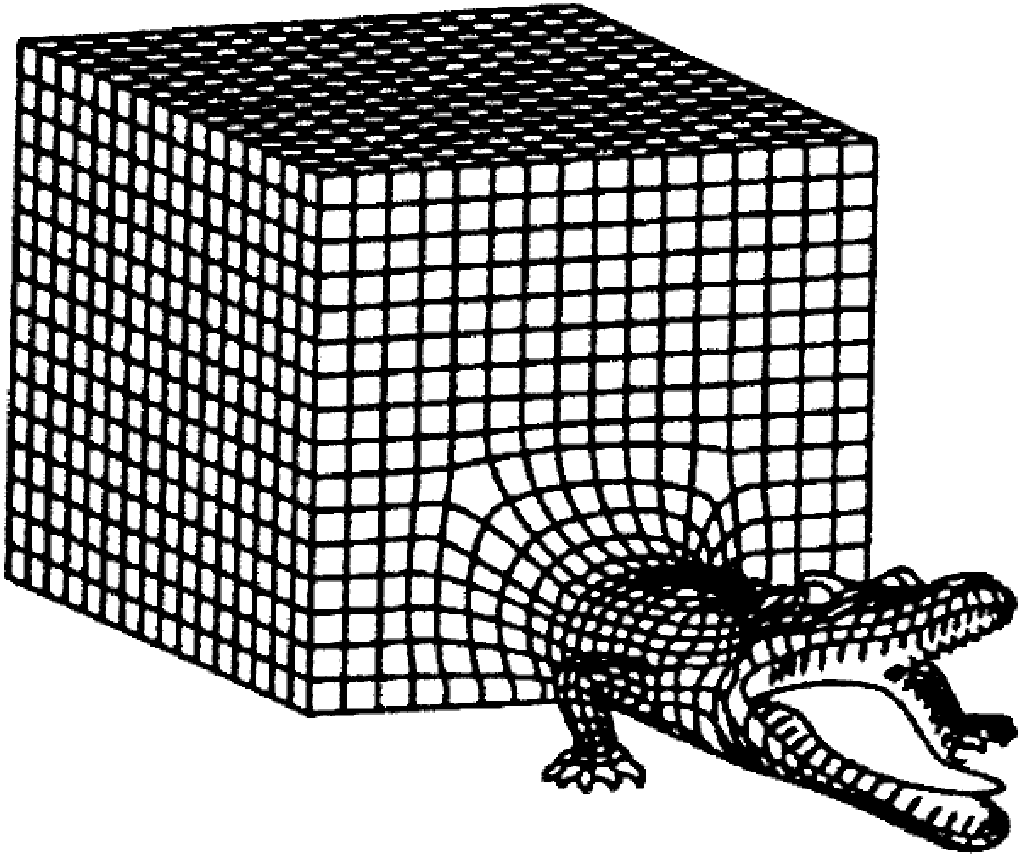


- Systematics for excited states are more significant

PROCEED WITH CAUTION



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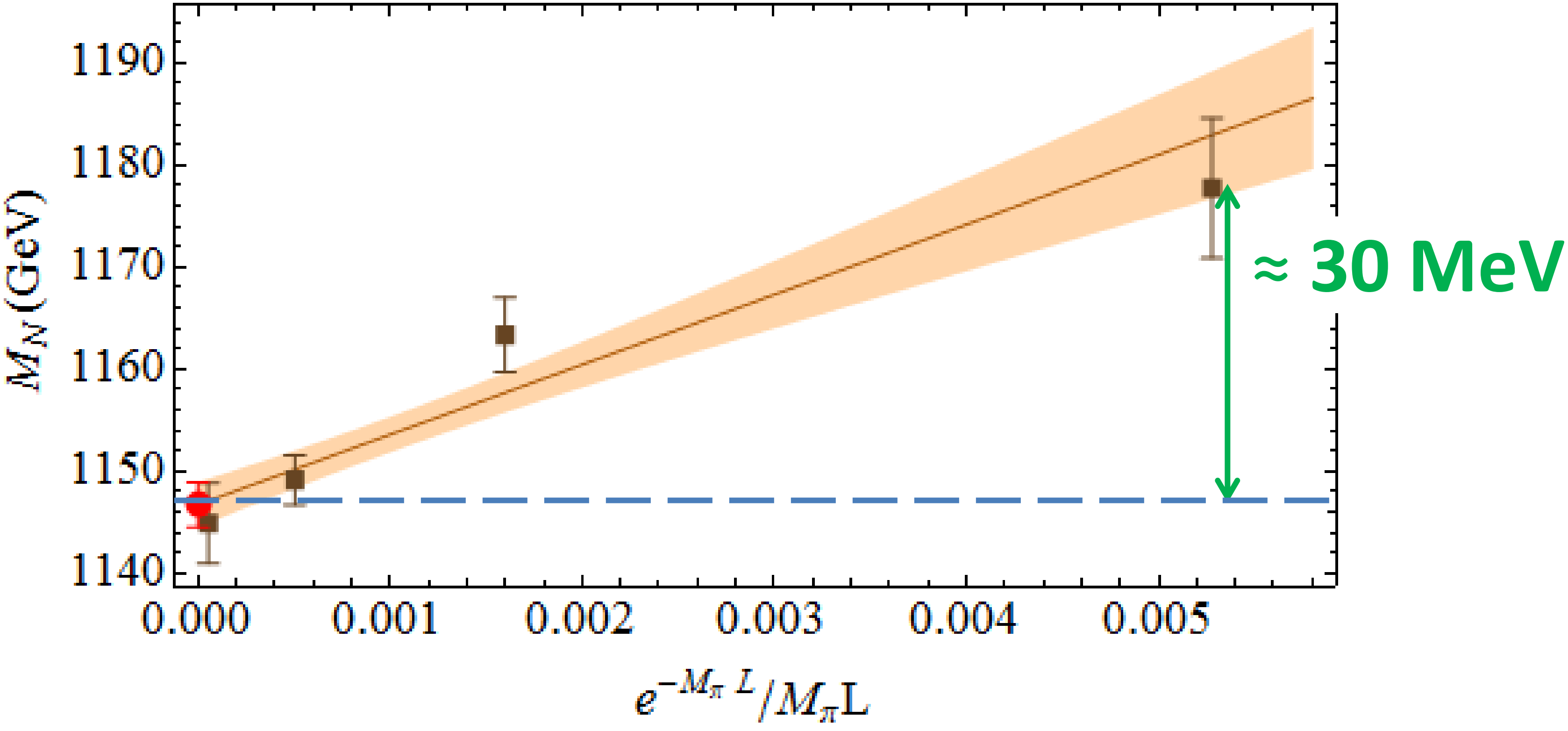
Volume Dependence

- 2+1f anisotropic lattices, $M_\pi \approx 390$ MeV, $L \approx 4, 3, 2.5, 2$ fm

$M_\pi L \approx 7.7, 5.8, 4.8, 3.9$



$$A_N(4\,\text{fm})/A_N(3\,\text{fm}) = 0.961(35)$$

NPLQCD, 1104.4101

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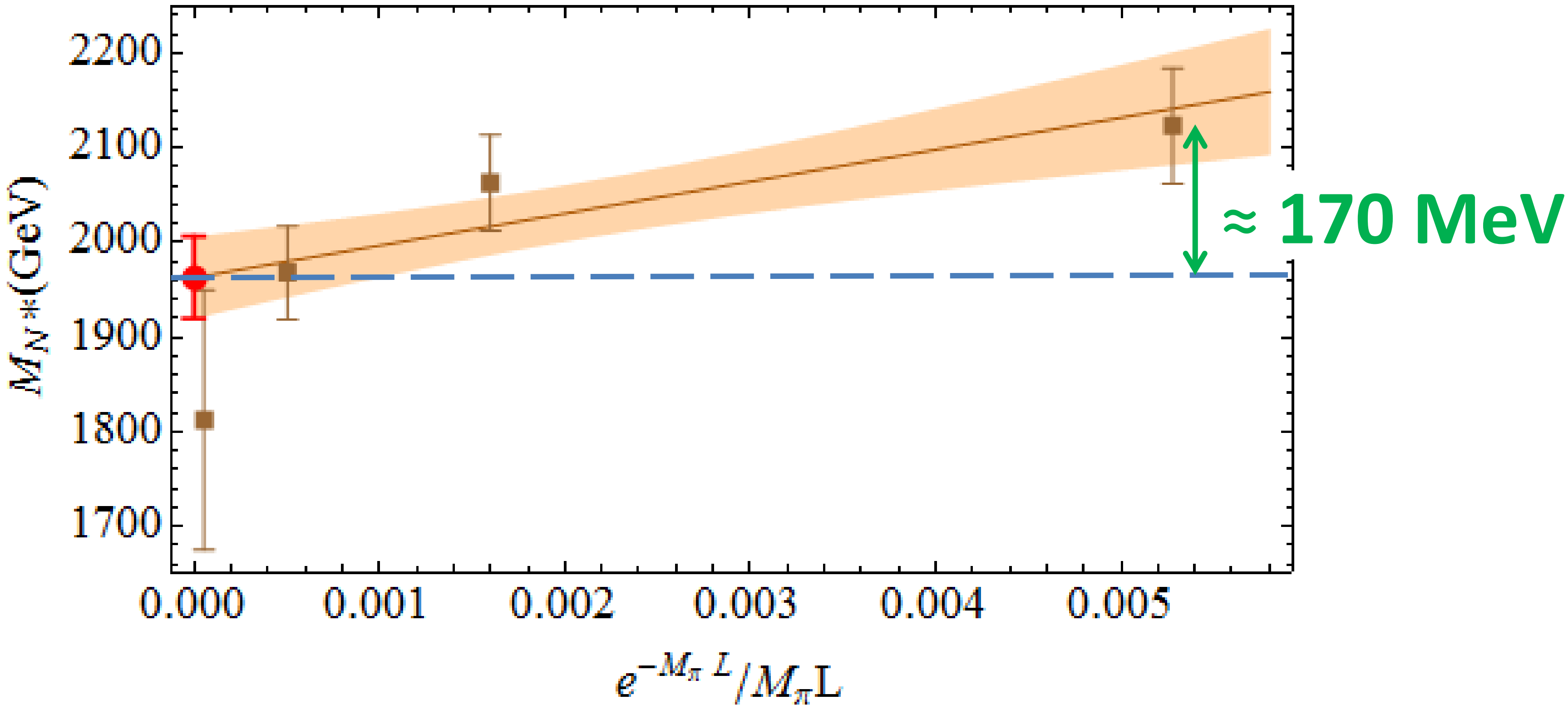


$A_N(4\,\mathrm{fm})/A_N(3\,\mathrm{fm}) = 0.961(35)$ $\quad$ $A_R(4\,\mathrm{fm})/A_R(3\,\mathrm{fm}) = 0.964(52)$

EM Form Factors

Form Factors

- Three-point function with interpolation operator *J*

$$C_{\text{3pt}}^{\Gamma,\mathcal{O}}(\vec{p},t,\tau)=\sum_{\alpha,\beta}\Gamma^{\alpha,\beta}\langle J_\beta(\vec{p},t)\,\mathcal{O}(\tau)\overline{J}_\alpha(\vec{p},0)\rangle$$

- The form factors are buried in the amplitudes

$$\Gamma^{(3),T}_{\mu,AB}(t_i,t,t_f,\vec{p}_i,\vec{p}_f)$$
$$= a^3\sum_n\sum_{n'}\frac{1}{Z_j}\frac{Z_{n',B}(p_f)Z_{n,A}(p_i)}{4E'_n(\vec{p}_f)E_n(\vec{p}_i)}e^{-(t_f-t)E'_n(\vec{p}_f)}e^{-(t-t_i)E_n(\vec{p}_i)}$$
$$\times\sum_{s,s'}T_{\alpha\beta}u_{n'}(\vec{p}_f,s')_\beta\langle N_{n'}(\vec{p}_f,s')\left|j_\mu(0)\right|N_n(\vec{p}_i,s)\rangle\overline{u}_n(\vec{p}_i,s)_\alpha$$

- Nucleon form factor (*n* = *n'* = 0)

$$\langle N\left|V_\mu\right|N\rangle(q)\;=\;\overline{u}_N(p')\left[\gamma_\mu F_1(q^2)+\sigma_{\mu\nu}q_\nu\frac{F_2(q^2)}{2m}\right]u_N(p)e^{-iq\cdot x}$$

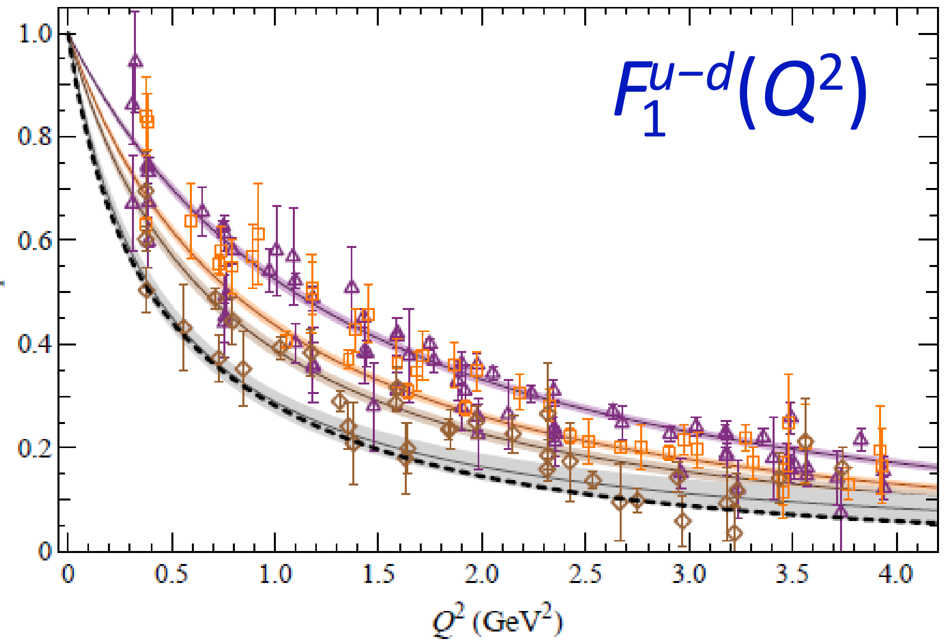
- Nucleon-Roper form factor (*n* = 0, *n'* = 1 or *n* = 1, *n'* = 0)

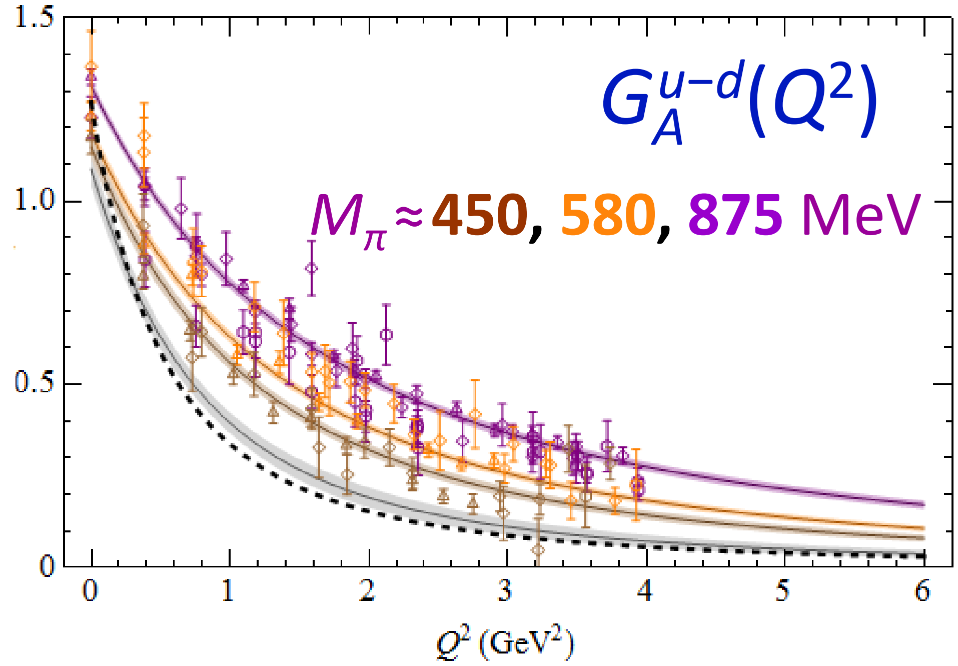
$$\langle N_2\left|V_\mu\right|N_1\rangle_\mu(q)\;=\;\overline{u}_{N_2}(p')\left[F_1(q^2)\left(\gamma_\mu-\frac{q_\mu}{q^2}\not{q}\right)+\sigma_{\mu\nu}q_\nu\frac{F_2(q^2)}{M_{N_1}+M_{N_2}}\right]u_{N_1}(p)e^{-iq\cdot x}$$

Nucleon Form Factors

- $N_f = 2+1$ anisotropic lattices, $M_\pi \approx$ **450**, **580**, **875** MeV

$$\overline{u}_N(p')\left[\gamma_\mu F_1(q^2) + \sigma_{\mu\nu}q_\nu \frac{F_2(q^2)}{2m}\right]u_N(p) \qquad \overline{u}_B(p')\left[\gamma_\mu\gamma_5 G_A(q^2) + \gamma_5 q_\nu \frac{G_P(q^2)}{2M_B}\right]u_B(p)$$



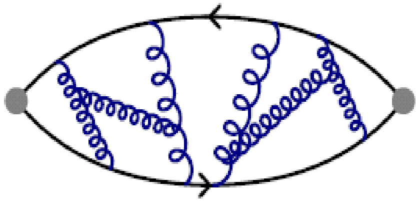


HWL et al., arXiv: 1005.0799, 1104.4319

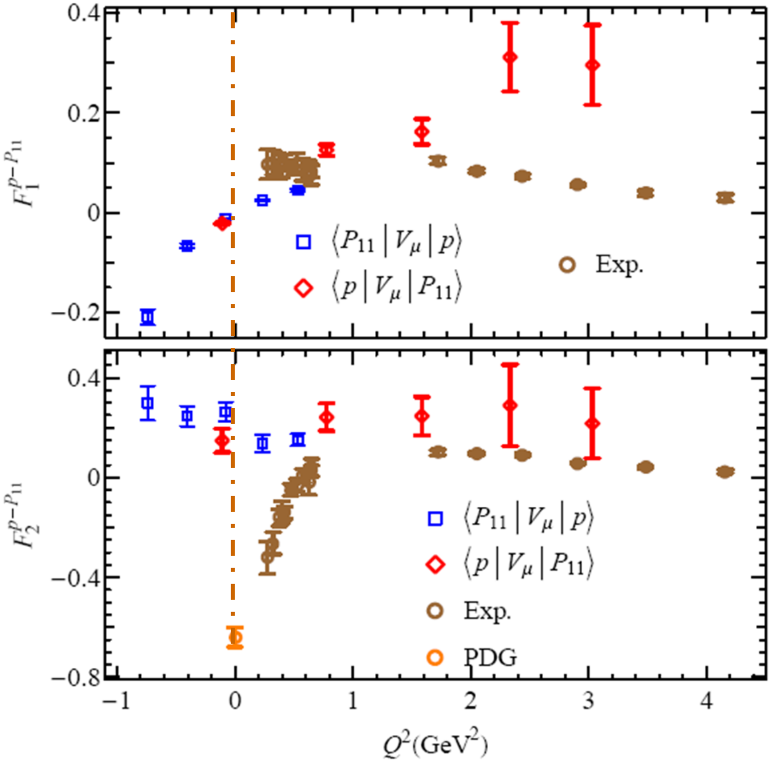
Nucleon-Roper Form Factors

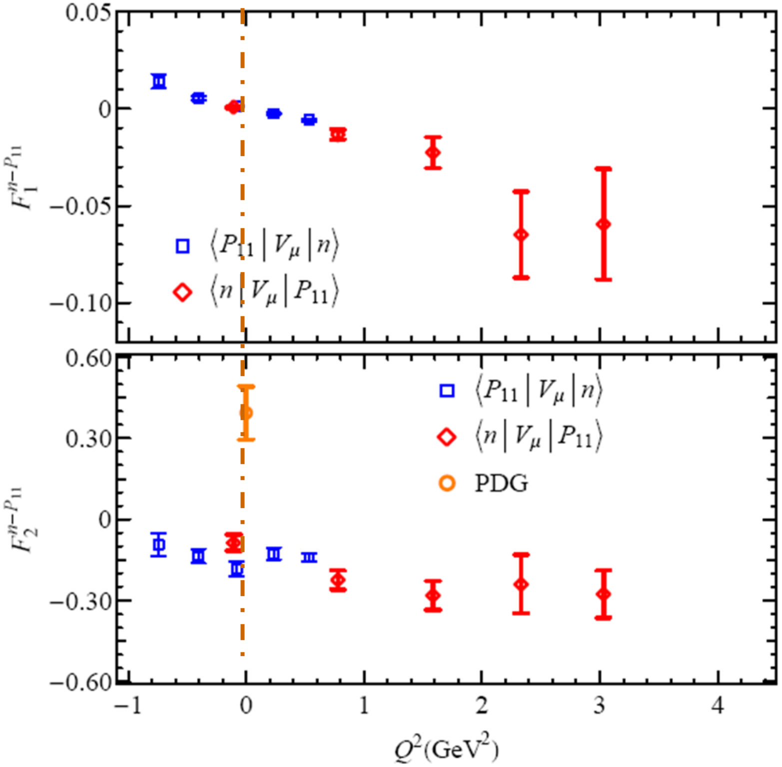
§ First *N*-*R* transition form factor from LQCD

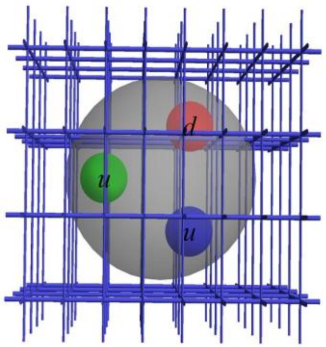
- 0f anisotropic clover ($a \approx 0.1$ fm, $\xi \approx 3$)



Proton-P_{11} **720-MeV Pion** **Neutron-P_{11}**





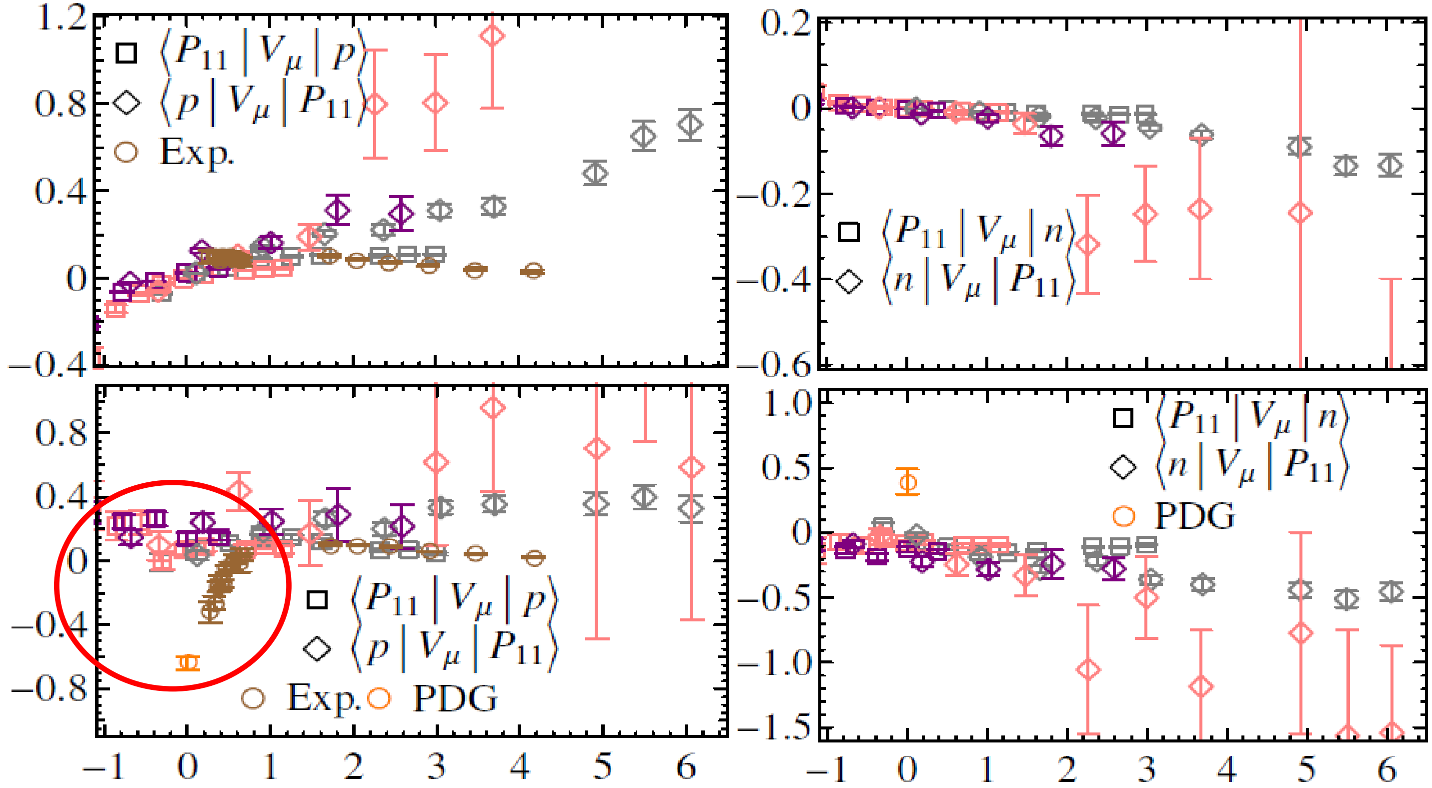


HWL et. al, Phys.Rev.D78:114508 (2008)

Nucleon-Roper Form Factors

§ Still quenched, more pion masses

- 0f anisotropic clover ($a \approx 0.1$ fm, $\xi \approx 3$)
- $M_\pi \approx$ 480, **720**, **1100** MeV



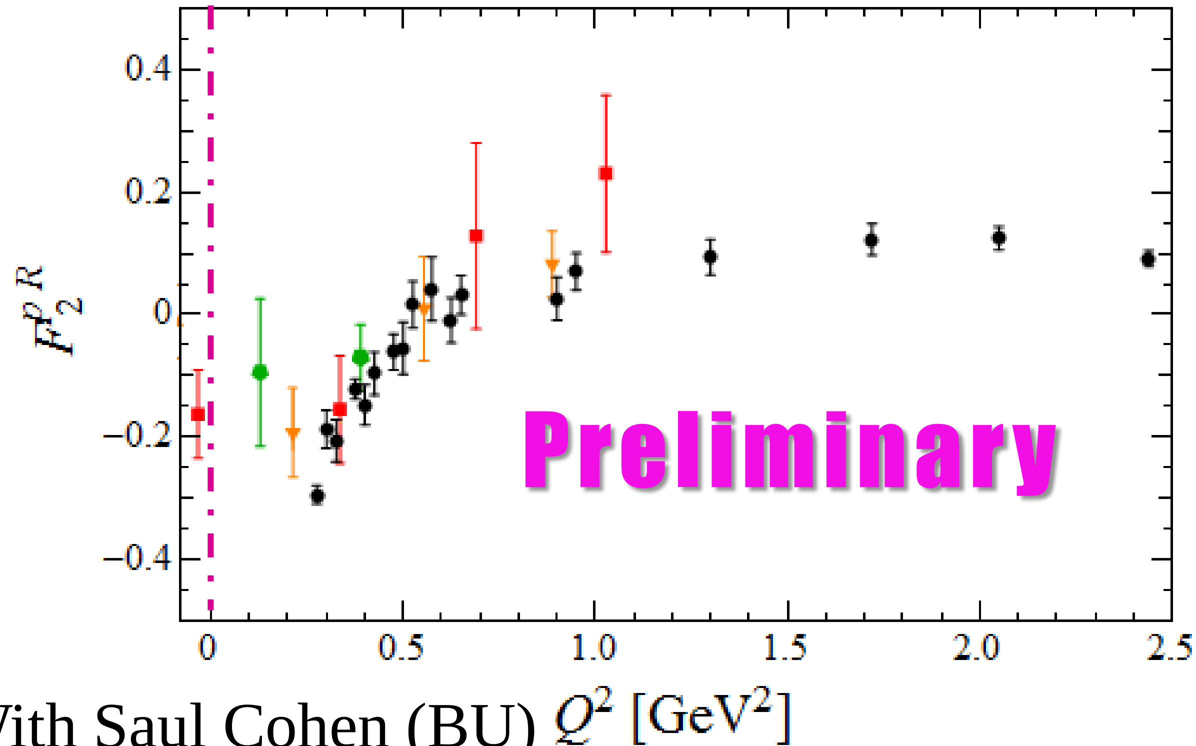
HWL et. al, PoS LATTICE2008:140 (2008)

Nucleon-Roper Form Factors

§ Turn on the quark loops in the sea

(L = 3, 2.5, 2.5 fm)

- 2+1f anisotropic clover with $M_\pi \approx$ **390**, **450**, **875** MeV



* With Saul Cohen (BU)

† Calculation performed using (NSF MRI PHY-0922770) Hyak cluster at U. Washington

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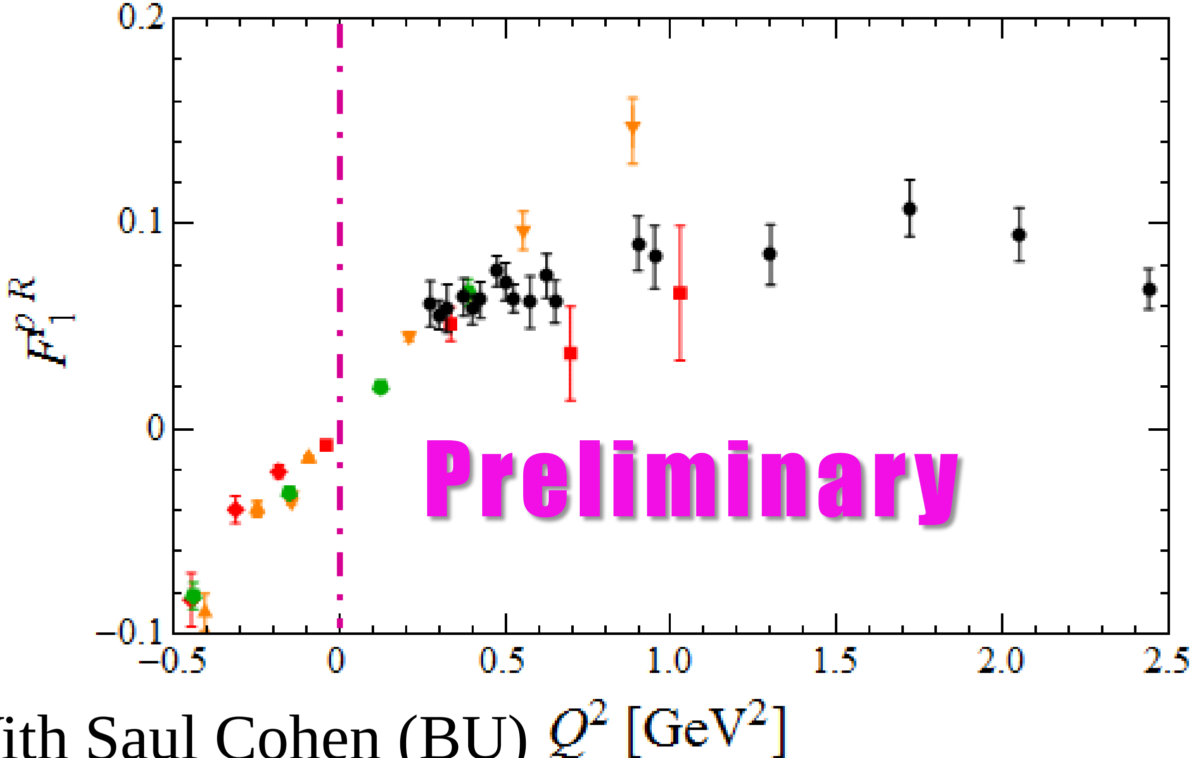

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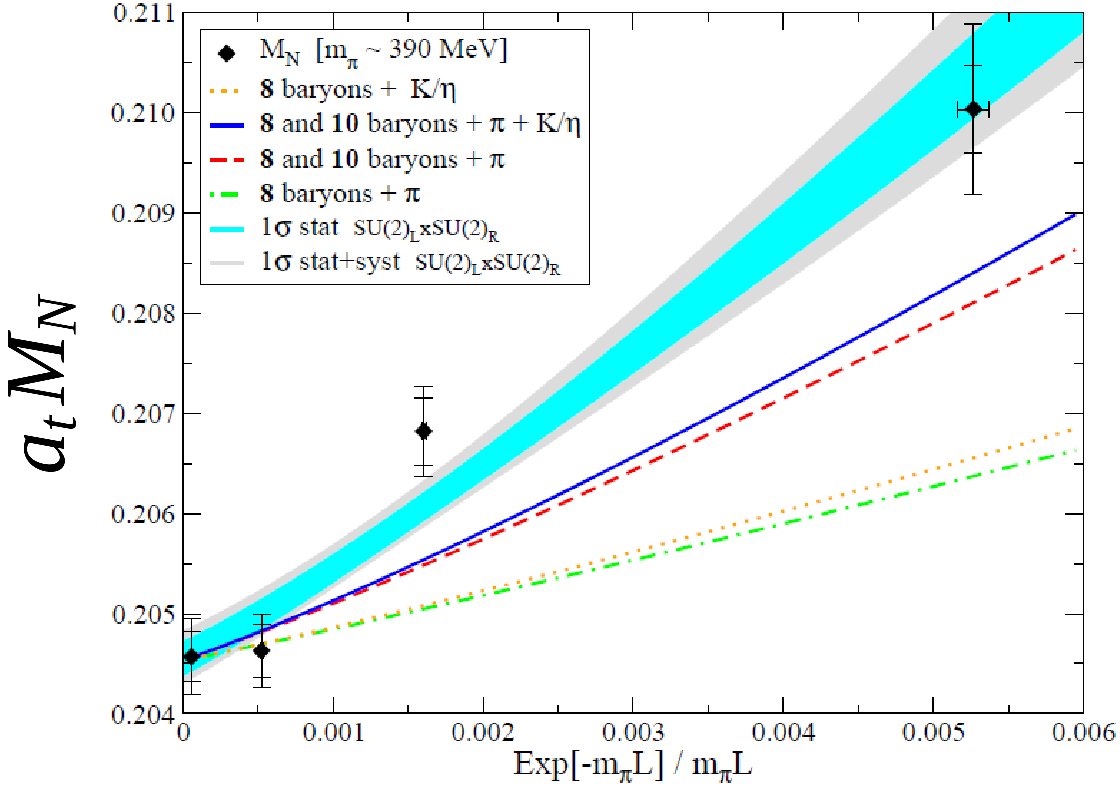
Axial Properties

Axial Couplings

§ NPLQCD, $m_\pi \approx 390$ MeV

$$|g_{\Delta N\pi}| = 2.80(18)(21)$$



Also see
C. Alexandrou,
this workshop

$$\delta M_N = \frac{9 m_\pi^3 g_A^2}{8\pi f_\pi^2} F_N^{(FV)}(m_\pi L) + \frac{m_\pi^3 g_{\Delta N\pi}^2}{\pi f_\pi^2} F_\Delta^{(FV)}(m_\pi L, \Delta_{\Delta N} L)$$

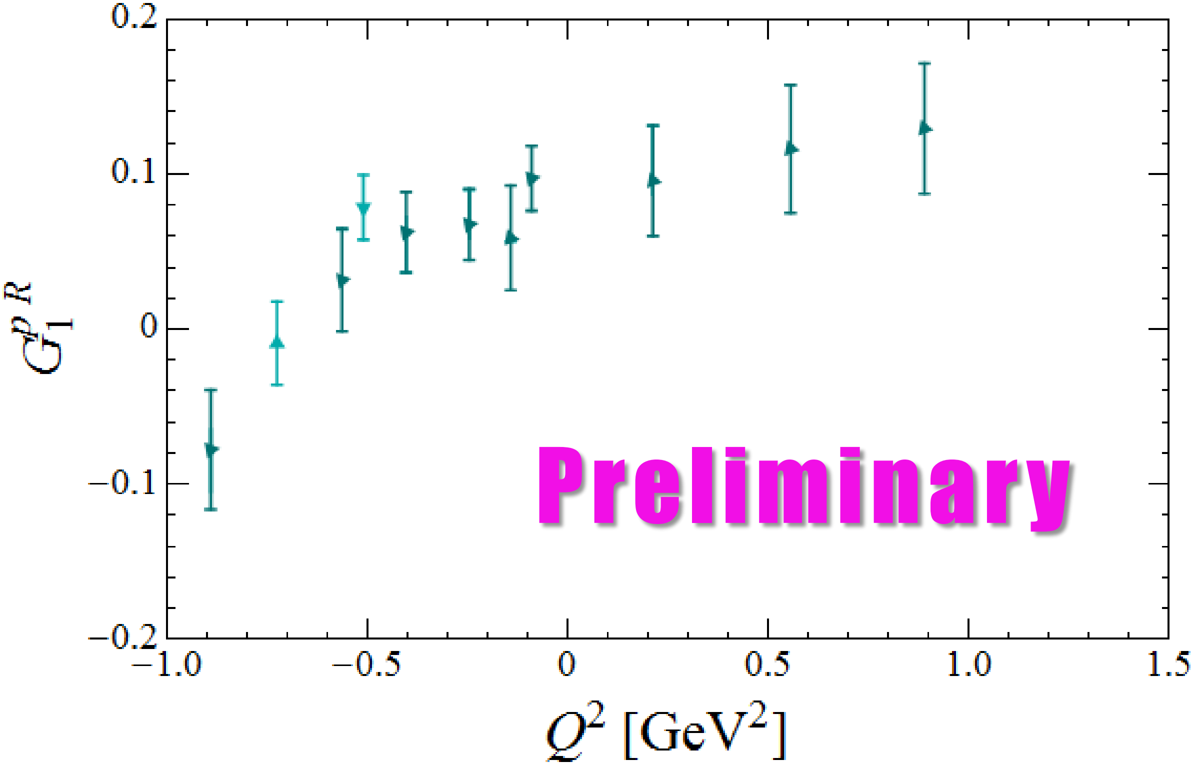
NPLQCD, 1104.4101

§ Similarly, $|g_{\Sigma^*\Lambda\pi}| = 2.21(16)(23)$ and $|g_{\Xi^*\Xi\pi}| = 2.49(23)(35)$

§ Find g_{A,N^*} and $g_{NN^*\pi}$ using a similar approach?

Axial Transition Form Factors

- Contains spin-structure information
- Combine with neutrino-nucleus cross sections to extract neutrino mass splittings and mixing angles
- 2+1 anisotropic lattices, $M_\pi \approx$ **390**, **450**, **875** MeV

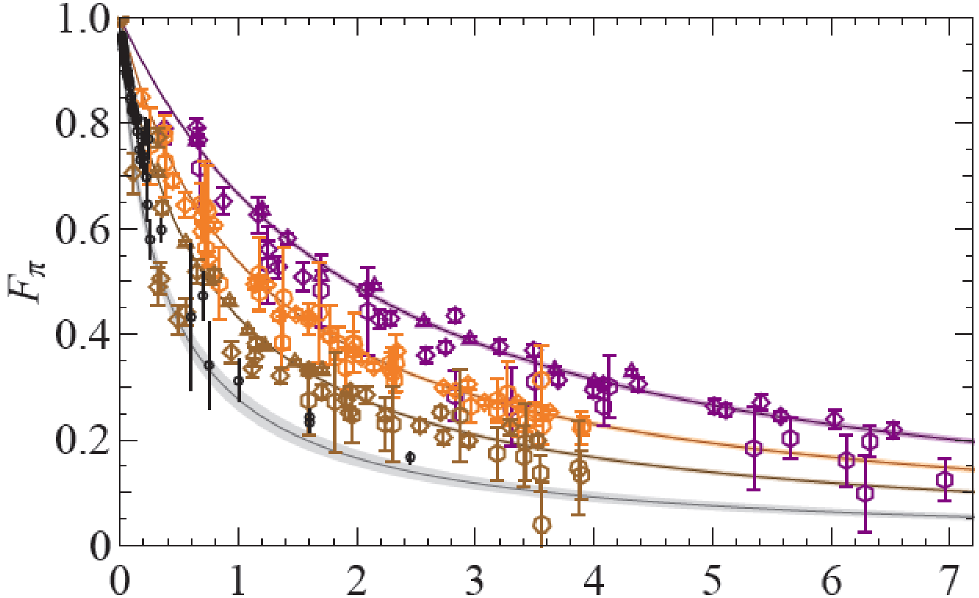


Large-Q^2 Prospects

Remarks

- Limited momenta on the lattice: $p^2 = (2\pi/L)^2\, n\, a^{-2}$
  - Larger n, finer a, …
- To get larger momentum, we use $O(ap) \approx 1$
  - Methodology for improving a traditional lattice calculation
  - Multiple lattice spacings to remove lattice artifacts

HWL et al., arXiv: 1005.0799, 1104.4319



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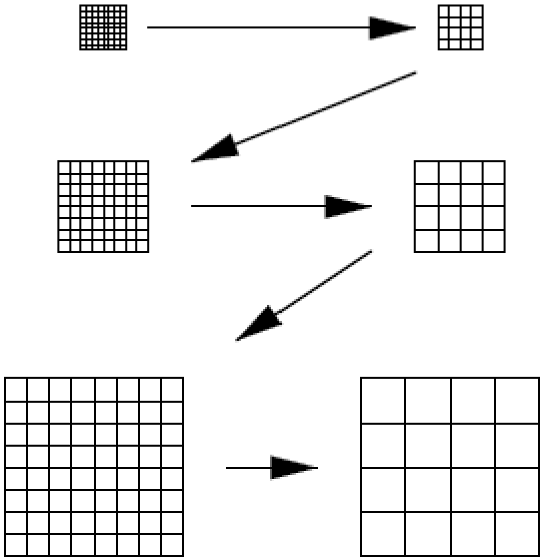
§ Possible future improvement

- Step-scaling through multiple lattice spacings and volumes
- Higher momentum transfer



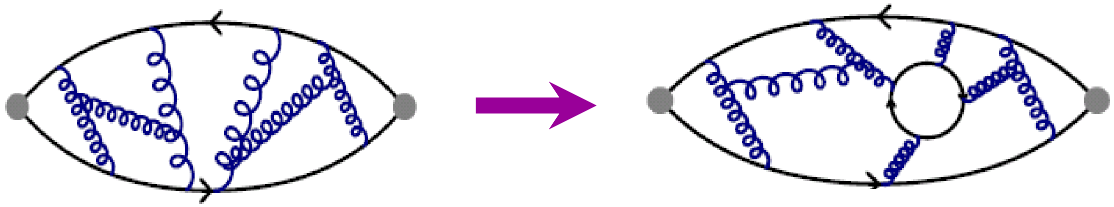
- Similar idea used for heavy quarks

HWL et al., Phys.Rev.D76:074506 (2007)

Summary and Outlook

§ Progress!

- From "quenched" to "dynamical"



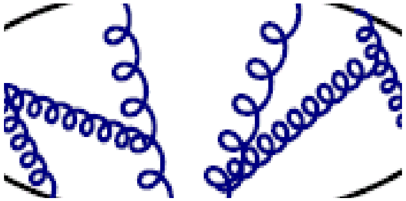
- Excited baryon resonances (spin identification by HSC)
- Transition axial couplings from direct or volume-dep. calculation
- p-P_{11} form factors
- Lighter pion mass, multiple lattice spacings, volumes, etc.

§ The future

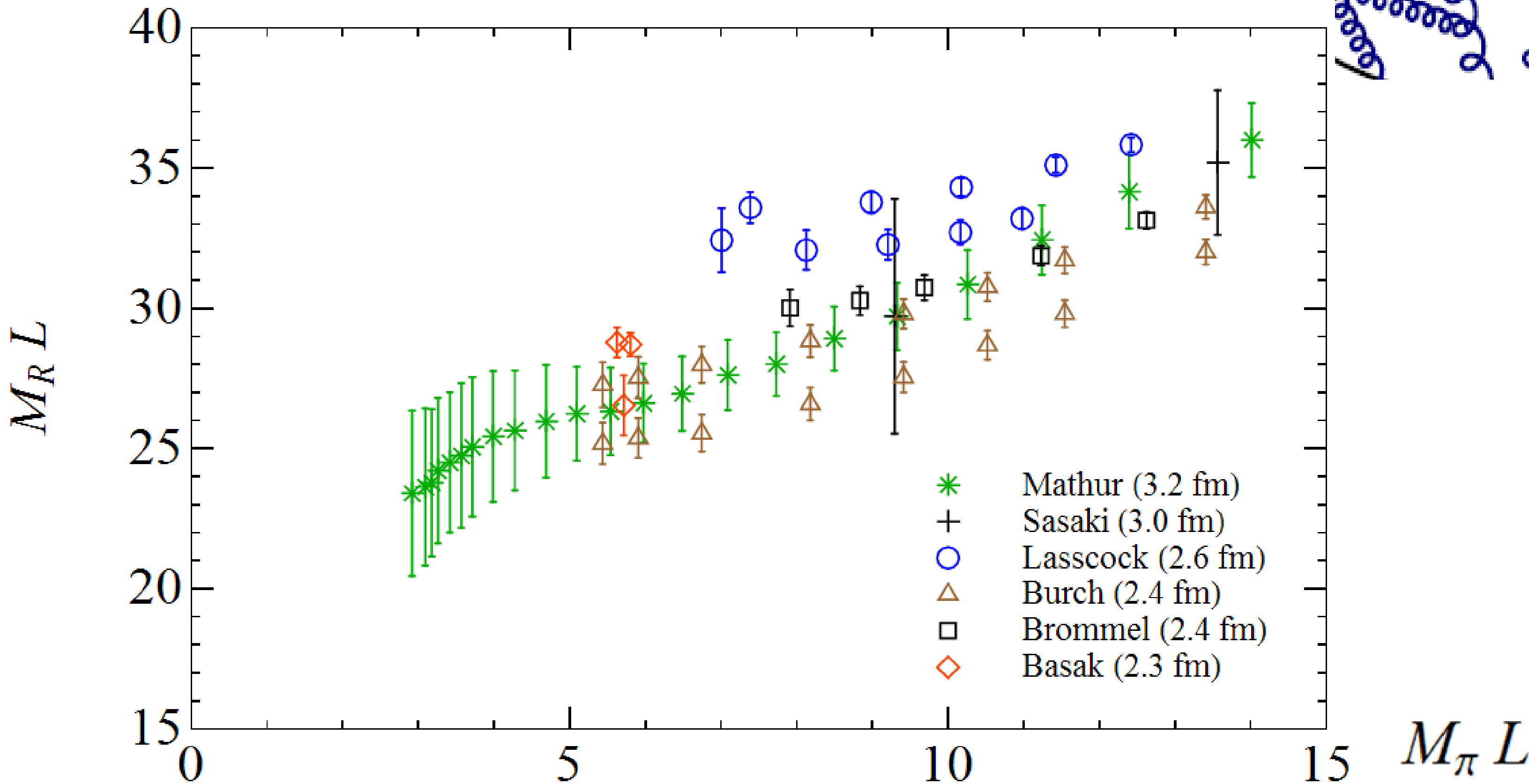
- Get a graduate student to increase the operator basis
  (plain single-point operator or fancy ones with little group)
- Check on the multiple-particle states

Backup Slides

Finite-Volume Dependence



- Size matters! More so for excited states



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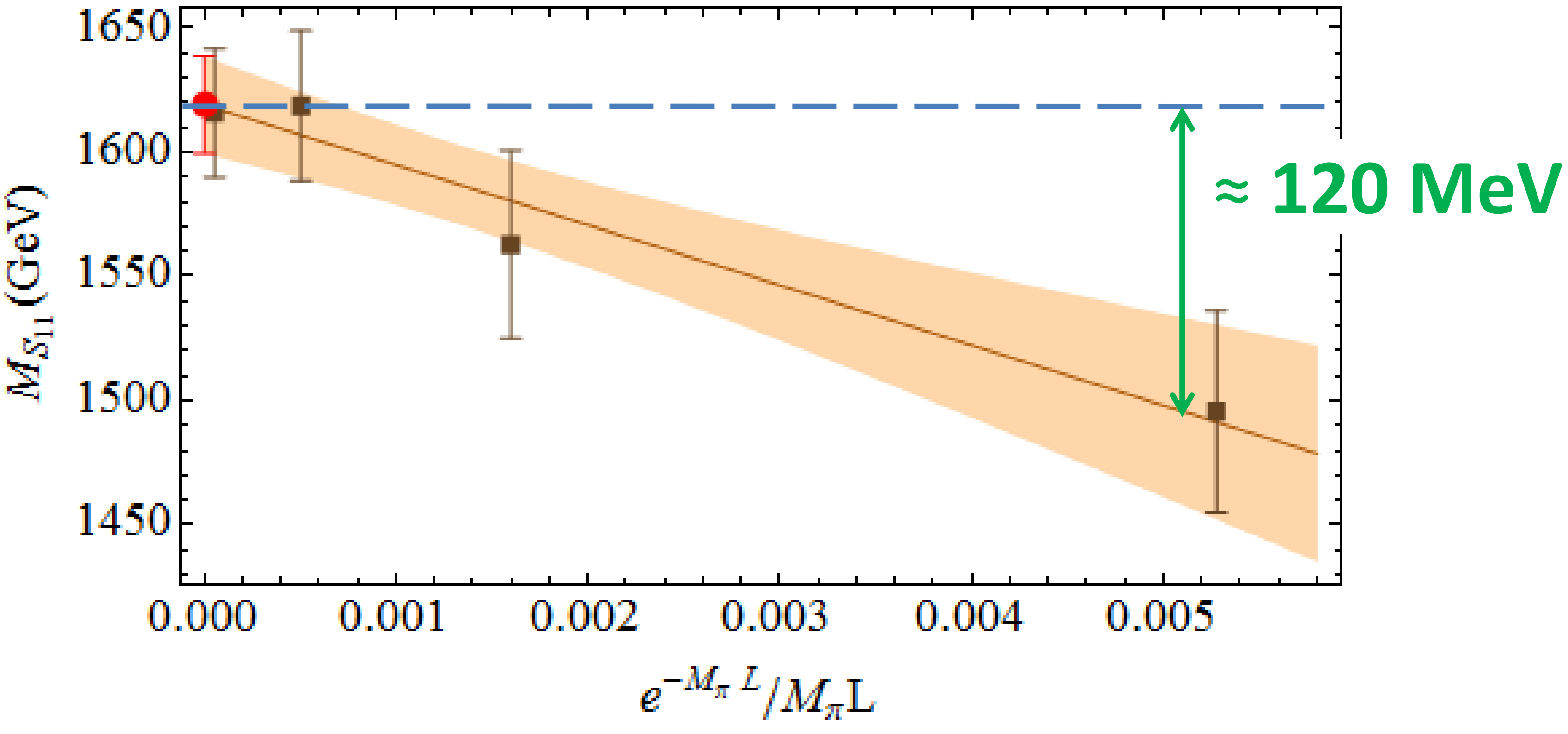

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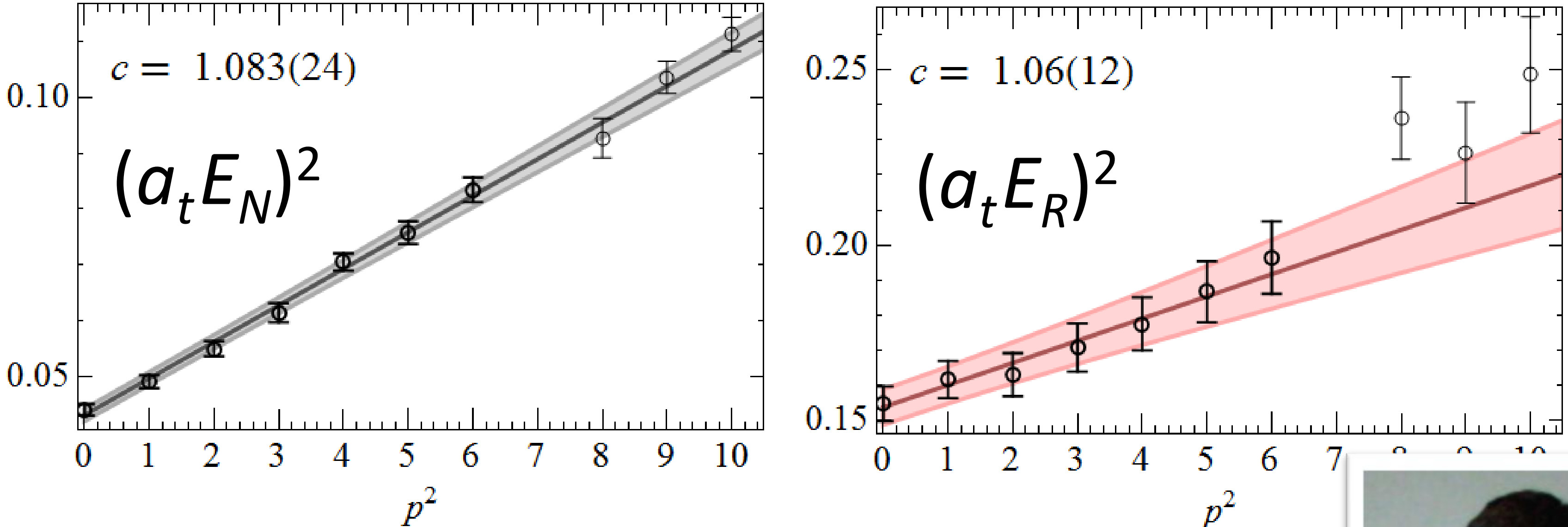

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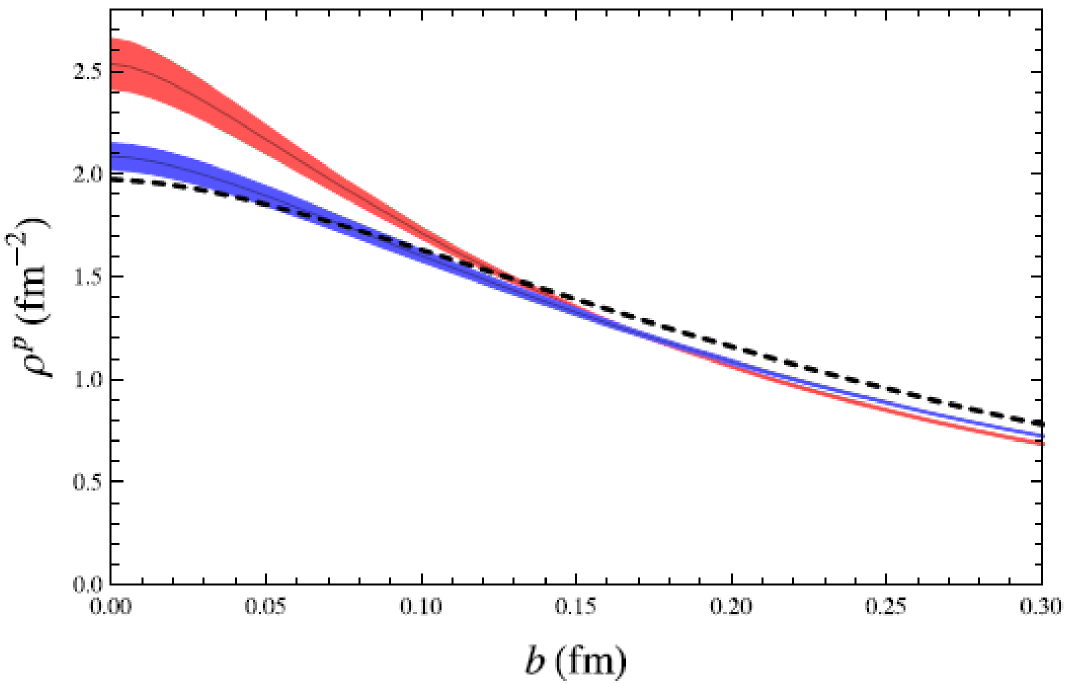
Transverse Charge Density

- Infinite-momentum frame definition

$$\rho(\mathbf{b}) \equiv \int \frac{d^2\mathbf{q}}{(2\pi)^2} F_1(\mathbf{q}^2) e^{i\mathbf{q}\cdot\mathbf{b}} = \int_0^\infty \frac{Q\,dQ}{2\pi} J_0(bQ) F_1(Q^2)$$

G. A. Miller, arXiv: 1002.0355

- How does high-Q^2 affect charge density?



- Red band uses lattice data ≤ 2.0 GeV^2
- Blue band uses lattice data ≤ 4.0 GeV^2

HWL et al., arXiv: 1005.0799